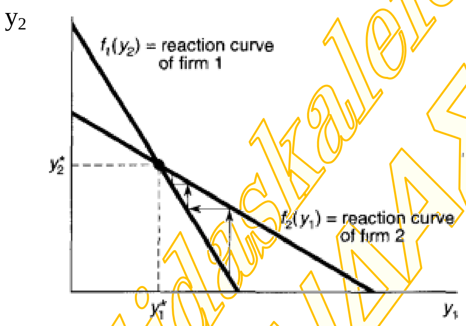
Duopoly – Cournot Equilibrium

There are two firms in the market, each face its cost function $C(q_1)$, $C(q_2)$ and produce a homogenous good with demand function $Q_D=f(P)$ or inverse demand function $P=f(Q_D)$.

The aggregate output (Total production) is given by : $Q_D=q_1+q_2$ so demand function can be written as $P=P(q_1+q_2)$

Each firm is maximizing its profits given its belief about the other firm's output level, we have 2 reaction functions, one for each firm. The intersection of the reaction functions is the Cournot-Nash equilibrium.



Reaction curves. The intersection of the two reaction curves is a Cournot-Nash equilibrium.

Example : Given a linear demand function $P=a-bQ$ or else $P = a - b(q_1+q_2)$

- Firm 's 1 maximization problem is :

$$\max \pi_1 = P\cdot q_1 - C(q_1)$$

$$\pi_1 = \left(a - b\left(q_1+q_2\right)\right)\cdot q_1 - C(q_1) \rightarrow \pi_1 = -b\cdot q_1^2 - b\cdot q_1\cdot q_2 + a\cdot q_1 - C(q_1) \quad so$$

$$f.o.c. \quad \frac{\partial \pi_1}{\partial q_1} = 0 \rightarrow -2b\cdot q_1 - b\cdot q_2 + a = MC(q_1) \Rightarrow$$

$$\Rightarrow reaction\ function \qquad q_1 = f\left(q_2\right) \quad (1)$$

- Firm 's 2 maximization problem is :

$$\max \pi_2 = P\cdot q_2 - C(q_2)$$

$$\pi_2 = \left(a - b\left(q_1+q_2\right)\right)\cdot q_2 - C(q_2) \rightarrow \pi_2 = -b\cdot q_2^2 - b\cdot q_1\cdot q_2 + a\cdot q_2 - C(q_2) \quad so$$

$$f.o.c. \quad \frac{\partial \pi_2}{\partial q_2} = 0 \rightarrow -2b\cdot q_2 - b\cdot q_1 + a = MC(q_2) \Rightarrow$$

$$\Rightarrow reaction\ function \qquad q_2 = g\left(q_1\right) \quad (2)$$

- By solving the system of equations (1) and (2) we find the equilibrium.

Exercise : In **a Cournot Duopoly** the market demand function is $y=100-2p$, the cost function of firm A is $C(y_A)=10+2y_A^2$ and the cost function of firm B is $C(y_B)=2+3y_B^2$.

a) What are the quantities of each firm and the price in market equilibrium? Draw the diagram.

b)What are the firm's profit levels

c) Assume that the two firms collude (**form cartel and monopolize the market**). What are the quantity and the price now?

d) Compare the total profits of questions (b) and (c). What can you conclude?

e) What is the market equilibrium under Stackelberg assumptions with firm A as the leader?

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Answer : The inverse demand function is : $P = 50 - \frac{y_A + y_B}{2}$

- Firm 's A maximization problem is :

$$\max \pi_A = P \cdot y_A - C(y_A)$$

$$\pi_A = \left(50 - \frac{y_A + y_B}{2}\right) \cdot y_A - \left(10 + 2y_A^{\ 2}\right) \to \pi_A = 50y_A - \frac{y_A^{\ 2}}{2} - \frac{y_A \cdot y_B}{2} - 10 - 2y_A^{\ 2} \quad so$$

$$f.o.c. \quad \frac{\partial \pi_A}{\partial y_A} = 0 \to 50 - y_A - \frac{y_B}{2} - 4y_A = 0 \Rightarrow 5y_A + \frac{y_B}{2} = 50$$

$$\Rightarrow \textit{reaction function} \qquad y_A = 10 - \frac{y_B}{10} \quad (1)$$

- Firm 's B maximization problem is :

$$\max \pi_B = P \cdot y_B - C(y_B)$$

$$\pi_B = \left(50 - \frac{y_A + y_B}{2}\right) \cdot y_B - \left(2 + 3y_B^{\ 2}\right) \to \pi_B = 50y_B - \frac{y_B^{\ 2}}{2} - \frac{y_A \cdot y_B}{2} - 2 - 3y_B^{\ 2} \quad so$$

$$f.o.c. \quad \frac{\partial \pi_B}{\partial y_B} = 0 \to 50 - y_B - \frac{y_A}{2} - 6y_B = 0 \Rightarrow 7y_B + \frac{y_A}{2} = 50$$

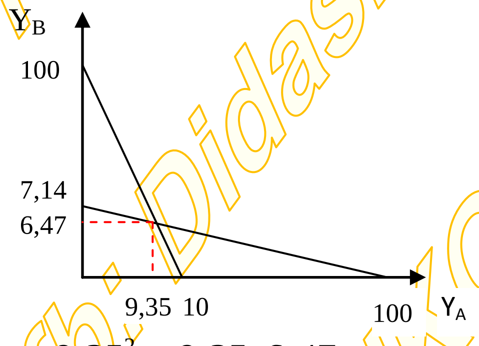
$$\Rightarrow \textit{reaction function} \qquad y_B = \frac{50}{7} - \frac{y_A}{14} \quad (2)$$

By solving the system of equations (1) and (2) we find the equilibrium

$y_B = 6,47$ *and* $y_A = 9,35$ So $y = y_A + y_B = 9,35 + 6,47 = 15,82$

and from the demand function $P = 50 - \frac{y_A + y_B}{2}$ follow P= 42,09

Diagram:



b)

$$\pi_A = 50 \cdot 9,35 - \frac{9,35^2}{2} - \frac{9,35 \cdot 6,47}{2} - 10 - 2 \cdot 9,35^2 = 208,69$$

$$\pi_B = 50 \cdot 6,47 - \frac{6,47^2}{2} - \frac{9,35 \cdot 6,47}{2} - 2 - 3 \cdot 6,47^2 = 143,81$$

c) The profit function under monopoly is given by

$$\pi = \left(50 - \frac{y_A + y_B}{2}\right) \cdot (y_A + y_B) - c_A - c_B \to$$

$$\to \pi = 50y_A + 50y_B - \frac{y_A^2}{2} - y_A \cdot y_B - \frac{y_B^2}{2} - c_A - c_B$$

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f.o.c

- $$\frac{\partial \pi}{\partial y_A} = 0 \to 50 - y_A - y_B - \left(10 + 2y_A^2\right)' = 0 \to 50 - y_A - y_B - 4y_A = 0 \to$$
  $$\to 5y_A + y_B = 50 \qquad (1)$$
- $$\frac{\partial \pi}{\partial y_B} = 0 \to 50 - y_A - y_B - \left(2 + 3y_B^2\right)' = 0 \to 50 - y_A - y_B - 6y_B = 0 \to$$
  $$\to y_A + 7y_B = 50 \qquad (2)$$

From (1) and (2) follows $y_A = 8.82, \quad y_B = 5.88, \quad y = y_A + y_B = 14.7$

so from the demand function $P = 50 - \frac{y_A + y_B}{2}$ follows P= 42,65

d) In duopoly $\pi = \pi_A + \pi_B = 208,69 + 143,81 = 352,5$

In monopoly

$$\pi = 50y_A + 50y_B - \frac{y_A^2}{2} - y_A \cdot y_B - \frac{y_B^2}{2} - \left(10 + 2y_A^2\right) - \left(2 + 3y_B^2\right) =$$

$$= 50 \cdot 8,82 + 50 \cdot 5,88 - \frac{8,82^2}{2} - 8,82 \cdot 5,88 - \frac{5,88^2}{2} - \left(10 + 2 \cdot 8,82^2\right) - \left(2 + 3 \cdot 5,88^2\right) =$$

$$= 355,65$$

The 2 firms by forming cartel are maximizing the total profit so they can split the joint profit.

e) Firm A is considered the quantity leader as a result it has the first-mover advantage. Firm B (follower) adjusts its produced quantities on formers choices, so $y_B = f(y_A)$

- Firm 's B Reaction function $y_B = \frac{50}{7} - \frac{y_A}{14}$ (1) (the same as Cournot)
- Firm 's A maximization problem is :

$$\pi_A = 50y_A - \frac{y_A^2}{2} - \frac{y_A \cdot y_B}{2} - 10 - 2y_A^2 \xrightarrow{y_B = \frac{50}{7} - \frac{y_A}{14}} 50y_A - \frac{5}{2}y_A^2 - \frac{1}{2}y_A\left(\frac{50}{7} - \frac{y_A}{14}\right) - 10 \to$$

$$\pi_A = \frac{650}{14}y_A - \frac{69}{28}y_A^2 - 10 \quad so$$

$$f.o.c. \quad \frac{\partial \pi_A}{\partial y_A} = 0 \to \frac{650}{14} - \frac{69}{14}y_A = 0 \Rightarrow y_A = 9,42$$

so from (1) $y_B = 6,47$

As a result, $y = y_A + y_B = 9,42 + 6,47 = 15,89$

and from the demand function $P = 50 - \frac{y_A + y_B}{2}$ follows P= 42,05

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